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Bubble trouble

When a stock market rises unsustainably, it can create a financial bubble that sooner or later will burst. **Tobias Preis** and **H Eugene Stanley** examine whether concepts from physics can be used to create a law describing exactly how such crashes occur

Wild fluctuations in the stock prices and currency exchange rates of every country have had a huge impact on the world economy and the personal fortunes of millions of us over the last few years. These instabilities have also had another, perhaps unintended, consequence – of thrusting the academic discipline of "econophysics" firmly into the limelight. But does a field that involves applying the concepts of statistical physics to economics really have anything important to contribute to discussions about the current economic crisis? Yes – absolutely – because finding laws describing fluctuations is the essence of statistical physics.

Physicists are not, of course, the first people to apply statistics to economics, with mathematicians also having contributed to the field for many years. One of their first significant breakthroughs came in 1900 when Louis Bachelier, working under the tutelage of the great French mathematician Henri Poincaré, published a PhD thesis in which he analysed real financial data. Bachelier claimed that a histogram of the changes in share price (measured over any period of time) forms a bell-shaped curve known as a Gaussian function, with very large fluctuations essentially never occurring. In other words, he believed that the chances of a serious crash occurring are almost zero. Such serious crashes are indeed very rare, but when they do occur, their effects can be devastating.

The model associated with Bachelier is often referred to as the "random walk" or "drunkard's walk" because he assumed that stock prices go up or down randomly by an amount that has a characteristic value. In recent years, however, econophysicists have been able to get their hands on a staggering quantity of real-time financial data, including the price, volume and timestamp of every transaction of every stock you can think of. Thanks to this information, which is now available in huge

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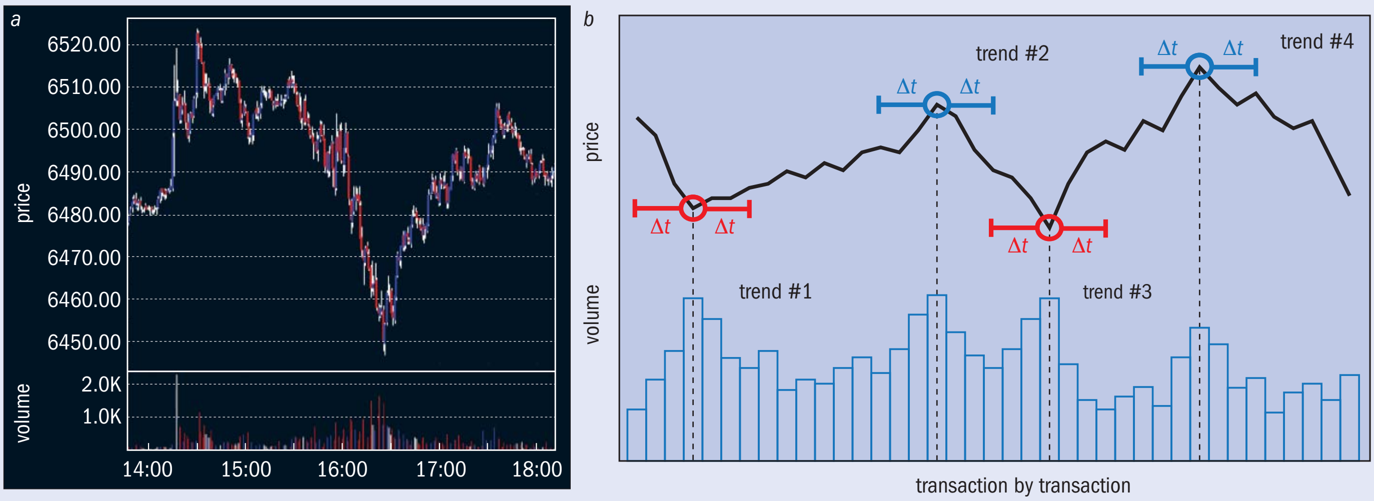
Preis will be giving a free online lecture at 4 p.m. BST on 12 May – see physicsworld.com/cws/channel/multimedia for more details

1 Rise and fall



(*a*) A small subset of the German DAX Future market comprising a few hours of trading on the afternoon of Friday 15 October 2010. The data can be analysed to obtain the price and volume of transactions as well as the time intervals between them, all with a resolution of 10 ms. (*b*) This graph shows local price maxima (blue circles) and local price minima (red circles), where these peaks and troughs are determined over a time window of $2\Delta t$. Positive trends start at the local minima and end at the local maxima – and vice versa for negative trends – with $\varepsilon = 0$ assigned to the start of each trend and $\varepsilon = 1$ to each end. In order to study the "switching processes" between trends – both before a trend switch and after – the subsequent volume sequences of identical length need to be considered.

financial databases, it is becoming widely accepted that the drunkard's walk fails to account for the very largest changes in stock price. These rare occurrences, which are collectively known as the "fat tails", correspond to events with huge price changes that are far more common than can be explained by Bachelier's model. However, as Bachelier's form of a Gaussian price-change histogram was still able to fit most of the available data, it led some mathematicians and economists to dismiss the fat tails as "outliers" or "tsunamis". Being so rare, they argued, the outliers can simply be ignored.

Expect the unexpected

Most physicists will find it incredulous that something that does not fit into a theory should be dismissed as merely an outlier. And econophysicists share that view too. But econophysicists have another trait – a reluctance to construct a theory before analysing as much relevant data as exist. Collecting significant amounts of data in traditional areas of physics often requires years of painstaking labour, but nowadays vast quantities of financial data are available, often free of charge.

In the late 1990s Paramaswaram Gopikrishnan and Vasiliki Plerou, who were then graduate students working at Boston University in the US, decided to analyse every transaction of every single stock in the major US markets. At that time, the analysis of such huge data sets was not as commonplace as it is today and required a significant upgrade to their university's computer system to complete the task. Using the extra computer resources, the two students constructed a histogram that displayed the number of times the stock market changed by a certain amount, plotted as a function of that amount. They did this by analysing 1000 different stocks each consisting of 200 000 data points.

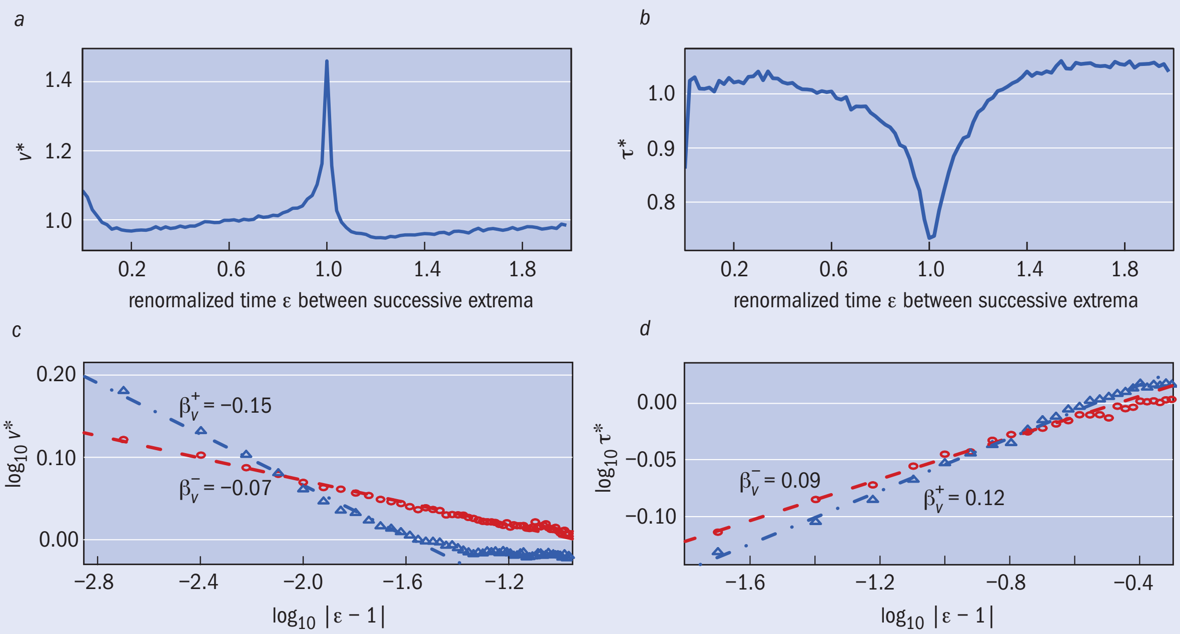
What Gopikrishnan and Plerou found was that large transactions are more common than they had expected, with the tail of their histogram not being Gaussian but following an "inverse quartic power law". This law means that if there is a probability p of, say, a \$5 price change occurring, then the probability of a \$10 price change is $p/2^4$, i.e. $p/16$. This inverse quartic law excels at describing the probability of very rare events, such as those occurring once every few decades. Events corresponding to 100 standard deviations, for example, have a probability of about 10^{-350} with a Gaussian model, but a far more realistic likelihood of 10^{-8} (i.e. one in a hundred million) with the inverse quartic law. This latter value is confirmed by real data. For example, Gopikrishnan and Plerou typically found two events corresponding to 100 standard deviations in a database of 200 million entries.

Although the inverse quartic law accounts for these incredibly rare events, as well as for more commonplace events, it is still not popular with some economists – particularly those who demand a complete theory that explains the causes before accepting empirical data. Physicists are not, however, accustomed to waiting for a fully formed theory before reporting new results. After all, the theoretical basis of many physical phenomena is often not understood for years after an experimental observation – and sometimes not even at all. To see this, you only have to look at the huge number of papers streaming out about high-temperature superconductivity despite the fact that we have no accepted theory for this phenomenon at all.

Shock and panic

The discovery of the inverse quartic law was a consequence of an intense study of a huge number of financial-transaction data. However, the law had one big flaw: it did not pay any regard to when the transactions took place. Individual price changes, in other words, were simply collected in the bins of a histogram and all "time ordering" of the data was lost. But can we find laws to describe this time ordering? It is a crucial question for econophysicists seeking to understand stock-market behaviour, which rises and then suddenly falls

2 Peaks and troughs



Whenever a stock market switches from a rising to a falling trend or vice versa, (*a*) the volume, v^*, of transactions peaks, while (*b*) the time between transactions, τ^*, reaches a minimum. These aggregated volume and "inter-trade" times were obtained by averaging all trends in data from the German DAX Future stock market, with the "renormalized time" on the *x*-axis running from the start of a trend ($\varepsilon = 0$) to the end ($\varepsilon = 1$). Price maxima and price minima coincide with peaks in the volume and dips in the inter-trade time. Analysis also reveals a power-law behaviour as seen from (*c*) this log–log plot of averaged transaction volumes versus ε both before reaching a switching point ($\varepsilon < 1$, circles) and after reaching a switching point ($\varepsilon > 1$, triangles). Power-law behaviour is also revealed by (*d*) this log–log plot of the averaged inter-trade times. β is the value of the slope.

in a manner that cries out for an explanation.

To analyse the time behaviour of financial data, what the current authors have done is to analyse massive data sets comprising three fluctuating quantities. These are: the price of each transaction; the number of exchanged shares in each transaction (i.e. the volume); and the time between each transaction and the next. Our goal was to find out if there are general laws to describe market behaviour near the "switching points" in the data – these are either local minima where the share price falls before starting to rise again (also known as an "uptrend") or local maxima where the price peaks before falling (a "downtrend"). In other words, are there regularities either just before or just after a switching point?

To find out, we analysed a time series of the German DAX Future stock market, which trades "future shares" of the top 30 German companies trading on the Frankfurt Stock Exchange (2010 *J. Stat. Phys*. **138** 431). It comprises 14 million transactions recorded with an incredible resolution of only 10 ms, which allows us to analyse in detail all "microtrends" occurring. We also looked at "macrotrends" over periods of up to 100 days by analysing 2.6 million daily closing prices of stocks in the S&P 500 share index in the US. What we first needed to do, however, was find a way of looking at all the data at once, without having to worry about the fact that the time between switching points can vary greatly from minutes to hours. Our solution was to "renormalize" the data by simply assigning a time $\varepsilon = 0$ to the start of an uptrend and $\varepsilon = 1$ to the end of an uptrend (figure 1). We also applied this methodology to negative trends, where $\varepsilon = 0$ and $\varepsilon = 1$ correspond to the start and end of a downtrend, respectively. Taken together, the two analyses let us look at positive and negative trends ranging in time over nine orders of magnitude (i.e. from 10^{-2} s to 10^{7} s).

Financial markets and their participants are well known to be hugely complex systems in which individual traders interact with one another through various mechanisms. For instance, traders are explicitly aware of all price fluctuations, and use this information when placing their individual orders – trying to guess when the price has a local maximum so they can sell at the best possible time, and trying to guess when the price is at a local minimum so they can buy at the best possible time. Although people have never quite been sure if this "panic hypothesis" is really true, we found plenty of evidence to back that view up. In particular, our analysis revealed that the volume of each transaction increases dramatically as the end of a trend is reached (figure 2*a*), while the time interval between each transaction drops (figure 2*b*). In other words, as prices start to rise or fall, stock is sold more frequently and in larger chunks. Traders become tense and panic because they are scared of missing a trend switch.

Based on the results of both databases, we also found to our surprise that there is a unique empirical power law quantifying both transaction volumes and inter-trade times in all the financial markets we analysed. Even more surprising, we found the same exponent on timescales varying over nine orders of magnitude, from 10 ms to 100 days (figure 2*c* and *d*). In other words, the formation of increasing and decreasing trends – bubbles and crashes – is scale-free in that the same law holds over up to at least nine orders of magnitude.

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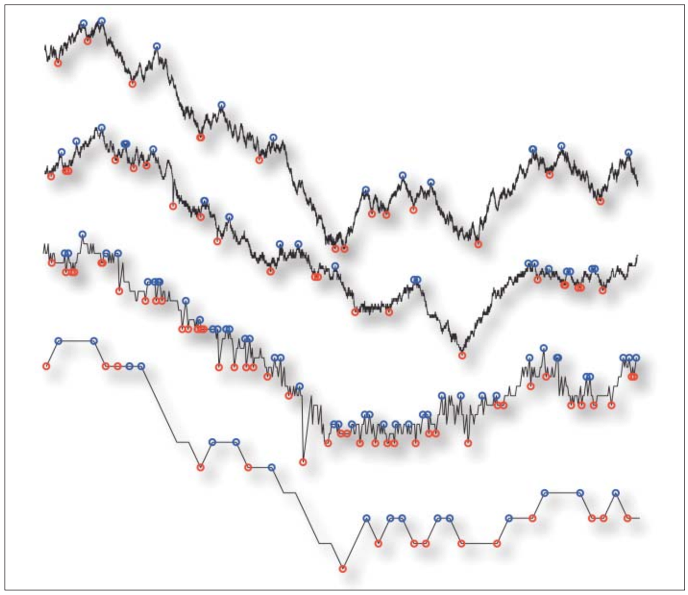

Scale-free market fluctuations Data from the German DAX Future stock-market index, with each curve showing just a tiny part of a data set consisting of 14 million transactions. The curves reveal how the rise and fall of share prices behave in a "self-similar" way, looking the same over whatever timescale you choose to pick – in this case over intervals of 1 day (top), 1 hour, 10 minutes and 1 minute (bottom). Local maxima and minima, or "switching points", are marked as blue and red circles.

Respect the data

Our results are consistent with the idea that the well-known catastrophic bubbles that occur over large timescales – such as the global financial crashes of 1929 and 2008 – may not be outliers. Instead, they are single dramatic representatives caused by the formation of increasing and decreasing trends on timescales varying over nine orders of magnitude from milliseconds to years. Just as the minor, tiny earthquakes that occur all the time turn out to be connected to the largest and most dramatic quakes, so it may be possible to learn more about the characteristics of dramatic crashes by studying the gargantuan amount of data concerning tiny crashes.

On typical trading days, economists who stick with traditional Gaussian models will have nothing to lose. However, they open themselves up to a false sense of security not unlike those who ignore very small earthquakes: unless you work in a seismic detection centre, you will be oblivious to the minor quakes occurring all the time. But huge earthquakes, when they do happen, can cause devastation. The 2008 Sichuan earthquake in China, for example, claimed nearly 70 000 lives, while, at the time of writing, more than 27 000 people have either died or remain unaccounted for after the earthquake in Japan in March.

Our experience might lead us to believe, naively, that a histogram of earthquake magnitudes will have a broad maximum for everyday earthquakes and a small sharp maximum for devastating earthquakes. Researchers studying actual seismic data, in fact, do not find this kind of "bimodal distribution" but rather a power law – the Gutenberg–Richter law. Yet while governments respect the real possibility of extremely rare huge earthquakes by requiring that buildings should be appropriately designed and constructed, it is not clear if policy makers equally respect the possibility of extremely rare huge financial shocks. ■