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# Fluctuation patterns in high-frequency financial asset returns

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**Abstract** – We introduce a new method for quantifying pattern-based complex short-time correlations of a time series. Our correlation measure is 1 for a perfectly correlated and 0 for a random walk time series. When we apply this method to high-frequency time series data of the German DAX future, we find clear correlations on short time scales. In order to subtract trivial autocorrelation parts from the pattern conformity, we introduce a simple model for reproducing the antipersistent regime and use alternatively level 1 quotes. When we remove the pattern conformity of this stochastic process from the original data, remaining pattern-based correlations can be observed.



**Introduction.** – Often the assumption is made that price dynamics of financial markets obey random walk statistics. However, real financial time series show deviations from this assumption [1–5], like fat-tailed price increment distributions [6–8]. Scaling behavior, short-time anti-correlated price changes and volatility clustering [9,10] are also well known and can be reproduced, *e.g.*, by a statistical model of the continuous double auction [11,12] or by various agent-based models [13–21]. Both price formation processes and cross correlations [22,23] between different stocks and indices have been studied with the intention to optimize asset allocation and portfolios. It is also known that stock markets display a reversion tendency after large price movements [24,25]. The rise of hedge fund industry in recent years and their interest in taking advantage of short-time correlations also boosted the analysis of the market microstructure, which is the study of the process of exchanging assets under explicit trading rules [26], and which is naturally studied and modeled intensively by the financial community [27–31] in order to minimize order execution costs.

In this letter, we study autocorrelations of financial market data in the anti-persistent short-time regime. For this purpose we analyze the randomness of financial markets employing specific conditional probability distribution functions, which reflect the main market response on given price impacts. According to common wisdom, the anti-persistence on short time scales is due to the bid ask bounce. In order to account for this effect, we introduce a simple stochastic model, in which the price is the sum of a random walk part and a second part describing the bid ask bounce. We show that beyond the correlations which are due to the bid ask bounce there are correlations in the fluctuation patterns, which we will call "complex correlations" in the following. In order to identify such complex correlations, we introduce a new method for quantifying pattern-based correlations of a time series on short time scales.

**Reproduction of the scaling behavior on short time scales.** – Scientific market modeling can only be based on price time series, which are the outcome of the trading decisions of the market participants comprising the "many particle system" of a financial market. The following analysis is based on historic price time series of the German DAX future contract (FDAX) traded at the European Exchange (EUREX), which is one of the world's largest derivatives exchanges. The time series, which is displayed in the inset of fig. 1, contains 2709952 trades recorded from 2 January 2007 to 16 March 2007.

A future contract is a contract to buy or sell a proposed underlying asset —in this case the German DAX index— at a specific date in the future at a specified price. The time series analysis of futures has the advantage that the prices are created by trading decisions alone. Contrarily, stock index data are derived from a weighted summation

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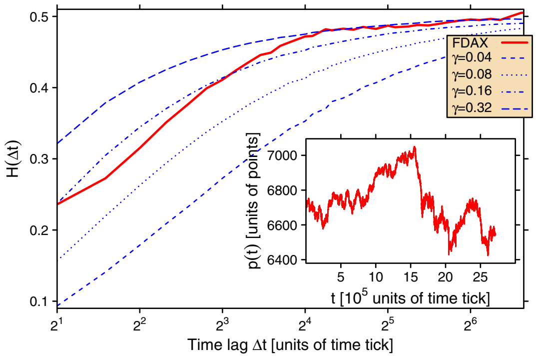


Fig. 1: (Color online) Hurst exponent $H(\Delta t)$ [32–35] in dependence of time lag $\Delta t$ calculated by the relationship $\langle |p(t+\Delta t)-p(t)|^q\rangle^{1/q} \propto \Delta t^{H_q(\Delta t)}$, with $q=2$ for the FDAX time series. Also shown is the Hurst Exponent for a synthetic anti-correlated random walk $p^*_\gamma(t)$ (ACRW) for various values of the random walk control parameter $\gamma$. The optimal value $\gamma=0.16$ is found by fitting the $\Delta t=1$ anti-correlation of the price time series to the one of the ACRW. Then, also the time lag dependence of the Hurst exponent is reasonably approximated. The FDAX time series is shown in the inset.

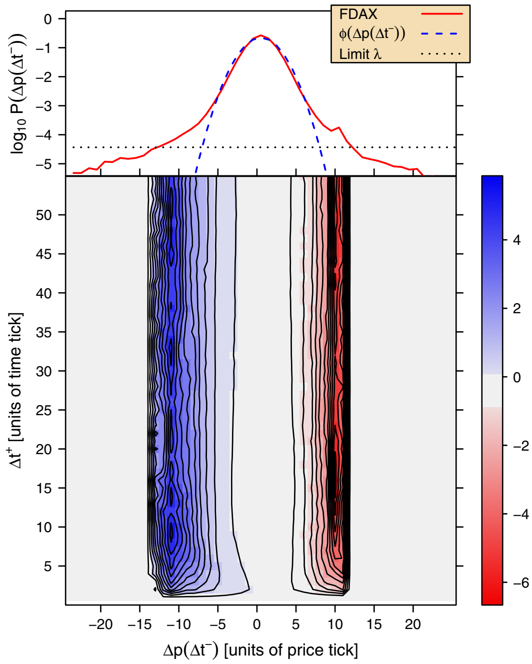


Fig. 2: (Color online) Probability distribution function (PDF) and conditional probability distribution function (CPDF) profile for the FDAX time series: In the upper part, the PDF $P(\Delta p(\Delta t^-))$ is shown semi-logarithmically for $\Delta t^-=10$. Additionally, a Gaussian least mean-square fit $\phi(\Delta p(\Delta t^-)) = u\exp(-v\Delta p^2)$ is provided in order to exhibit the fat-tailed nature of the price change distributions. The CPDF is only presented for price movements $\Delta p$, occurring with probability larger than a threshold value $\lambda$ indicated in the top part. In the bottom part, the price changes are analyzed conditionally. The color code gives the conditional expectation value $\langle p(t+\Delta t^+)-p(t)|\Delta p(\Delta t^-)\rangle_t$ in dependence of $\Delta t^+$ and $\Delta p(\Delta t^-)$.

of stock prices. With a large liquidity and inter-trade waiting times down to $10^{-2}$ seconds, an impressive data base is available, containing the transaction prices, the volumes, and the appropriate time stamps. Let $p(t)$ be the transaction price at time $t$, which is a discrete variable $t=1,2,\ldots,T$. As shown in fig. 1, the time-lag–dependent Hurst exponent $H(\Delta t)$ indicates an anti-persistent behavior of financial data sets on short time scales. The Hurst exponent is calculated by a local derivative of the mean-square displacement, *i.e.*, the relationship $\langle (p(t+\Delta t)-p(t))^2\rangle \propto \Delta t^{2H(\Delta t)}$ is used. This is a trivial consequence of the negative autocorrelation of price time series at time lag 1, caused by the non-zero bid-ask spread —the gap between the best available offer and the best available demand in an order book, which stores offers and demands during the trading process [19]. These jumps around the spread can be added synthetically to a random walk. Let $p^*_\gamma(t)$ be the time series of the synthetically anti-correlated random walk created in a Monte Carlo simulation through $p^*_\gamma(t)=a_\gamma(t)+b(t)$. With probability $\gamma\in[0;1/2]$ the expression $a_\gamma(t+1)-a_\gamma(t)=+1$ will be applied and with probability $\gamma$ a decrement $a_\gamma(t+1)-a_\gamma(t)=-1$ will occur. With probability $1-2\gamma$ the expression $a_\gamma(t+1)=a_\gamma(t)$ is used. The stochastic variable $b(t)$ models the bid-ask spread and can take the value 0 or 1 in each time step, each with probability 1/2. Thus, by changing $\gamma$, the characteristic time scale of process $a_\gamma$ in comparison to process $b$ can be modified. As shown in fig. 1, the strength of anti-persistence is controllable. For $\gamma=0.16$ the anti-persistence of the FDAX data is reproduced resulting also in a reasonable agreement with the observed time dependence of the Hurst exponent.

**Probability distribution function.** – A fat-tailed overall probability distribution function (PDF) of price changes is shown exemplarily for a time interval $\Delta t^-=10$ in the upper part of fig. 2 and for a time interval $\Delta t^-=45$ in the upper part of fig. 3. In order to examine the randomness of future price movements in the time interval $\Delta t^+$ in dependence of previous price impacts $\Delta p$ in the time interval $\Delta t^-$ one can look at conditional probability distribution functions (CPDF).

In the lower part of fig. 2, this conditional expectation value $\langle p(t+\Delta t^+)-p(t)|p(t)-p(t-\Delta t^-)\rangle_t$ in dependence of the time interval $\Delta t^+$ and the price jumps $\Delta p(\Delta t^-)$ for $\Delta t^-=10$ is presented. A tendency to counterbalance jumps can be clearly identified. On average a price reduction of 10 price ticks is, *e.g.*, counteracted by about 5 price ticks within 10 transactions. These results can be reproduced only qualitatively by the trivial random walk model introduced before. Trivially, process $b$ can counteract by maximally 1 tick. Qualitatively, the counteracting tendency is the same and is due to the anti-correlation of the time series for lag 1. However, the modified random walk has no fat tails by construction, reducing the counteracting effect. Also for other values $\Delta t^-\in[1;100]$ the

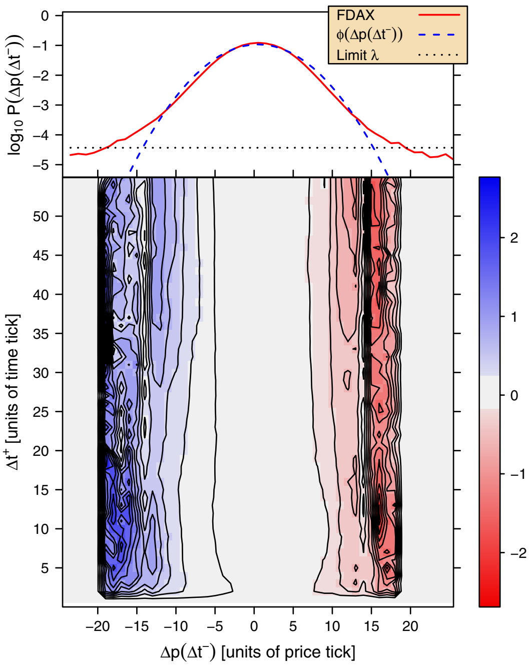


Fig. 3: (Color online) Probability distribution function (PDF) and conditional probability distribution function (CPDF) profile for FDAX time series for $\Delta t^- = 45$. In contrast to fig. 2 with $\Delta t^- = 10$, the PDF is broader and also the corresponding CPDF in the lower part shows more details. A smaller counter-acting tendency can be observed for this time interval. In the online version one can find an animation (`cpdf.gif`, 2.8 MB), which illustrates the dependency of $\Delta t^-$ in detail.

most significant counter-movements are present in the non-Gaussian tails.

**Pattern conformity.** – This investigation supports the assumption that the CPDF profile of financial market data on short time scales is influenced but not completely determined by the anti-correlation at time lag one. If complex correlations exist on these time scales one has to find a sophisticated observable to quantify them. The existence of such correlations implies that market participants —human traders and most notably automated trading algorithms— react to a given time series pattern just like to comparable patterns in the past. On medium and large time scales, this is the basic assumption of the controversially discussed technical analysis. However, on tick-by-tick basis, the effect of algorithmic trading is larger. To quantify the additional correlations, we define a general pattern conformity observable, which is not limited to the application to financial market time series.

The aim is to compare the current reference pattern of time interval length $\Delta t^-$ with all previous patterns in the time series $p(t)$. The current observation time shall be denoted by $\hat{t}$, then the reference interval is given by $[\hat{t} - \Delta t^-; \hat{t}\,)$. The forward evolution after this current reference interval —the distance to $\hat{t}$ is expressed by $\Delta t^+$— is compared with the prediction derived from historical patterns. As the volatility is not constant in time, all

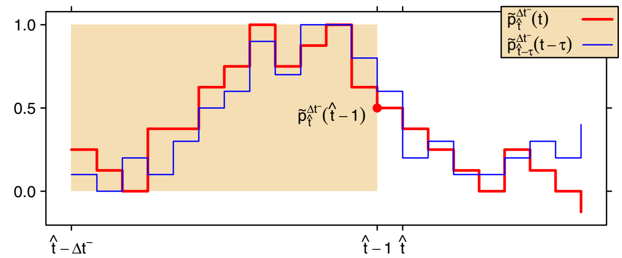


Fig. 4: (Color online) Schematic visualization of the pattern conformity calculation mechanism. The normalized reference pattern $\tilde{p}_{\hat{t}}^{\Delta t^-}(t)$ and the by $\tau$ shifted comparison pattern $\tilde{p}_{\hat{t}-\tau}^{\Delta t^-}(t-\tau)$ have the maximum value 1 and the minimum value 0 in $[\hat{t} - \Delta t^-; \hat{t}\,)$, as illustrated by the filled rectangle. For the pattern conformity calculation, it will be checked for each time interval $\Delta t^+$ starting at $\hat{t}$ whether reference and comparison pattern are above or below the last value of the reference pattern $\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1)$. If both are above or below this level, then $+1$ is added to the non-normalized pattern conformity. If one is above and the other below, then $-1$ is added.

comparison patterns have to be normalized with respect to the current reference pattern. For this reason, we use the true range —the difference between high and low. Let $p_h(\hat{t}, \Delta t^-)$ be the maximum value of a pattern of length $\Delta t^-$ at time $\hat{t}$ and analogously $p_l(\hat{t}, \Delta t^-)$ be the minimum value. We construct a modified time series, which is true range adapted in the appropriate time interval, through

$$\tilde{p}_{\hat{t}}^{\Delta t^-}(t) = \frac{p(t) - p_l(\hat{t}, \Delta t^-)}{p_h(\hat{t}, \Delta t^-) - p_l(\hat{t}, \Delta t^-)}, \qquad (1)$$

with $\tilde{p}_{\hat{t}}^{\Delta t^-}(t) \in [0;1] \;\; \forall \;\; t \in [\hat{t} - \Delta t^-; \hat{t}\,)$, as illustrated in fig. 4. At this point, the fit quality $Q_{\hat{t}}^{\Delta t^-}(\tau)$ between the current reference sequence $\tilde{p}_{\hat{t}}^{\Delta t^-}(t)$ and a comparison sequence $\tilde{p}_{\hat{t}-\tau}^{\Delta t^-}(t-\tau)$ for $t \in [\hat{t} - \Delta t^-; \hat{t})$ has to be determined by a least mean-square fit through

$$Q_{\hat{t}}^{\Delta t^-}(\tau) = \sum_{\theta=1}^{\Delta t^-} \frac{\left(\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-\theta) - \tilde{p}_{\hat{t}-\tau}^{\Delta t^-}(\hat{t}-\tau-\theta)\right)^2}{\Delta t^-}, \qquad (2)$$

with $Q_{\hat{t}}^{\Delta t^-}(\tau) \in [0,1]$ as a result of the true range adaption.

In order to quantify the value of reference and comparison pattern relative to the reference point $\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1)$ at time $\Delta t^+$ after $\hat{t}$, one can define

$$\omega_{\hat{t}}^{\Delta t^-}(\tau, \Delta t^+) = \left(\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1+\Delta t^+) - \tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1)\right) \times \left(\tilde{p}_{\hat{t}-\tau}^{\Delta t^-}(\hat{t}-\tau-1+\Delta t^+) - \tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1)\right), \qquad (3)$$

as motivated in fig. 4. In the example shown in fig. 4, $\omega_{\hat{t}}^{\Delta t^-}(\tau, \Delta t^+)$ is larger than 0 for all possible $\Delta t^+$, as $\tilde{p}_{\hat{t}-\tau}^{\Delta t^-}(\hat{t}-\tau-1+\Delta t^+)$ is below and $\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1+\Delta t^+)$ is also below the reference point $\tilde{p}_{\hat{t}}^{\Delta t^-}(\hat{t}-1)$. In such cases, the pattern conformity observable should be increased in

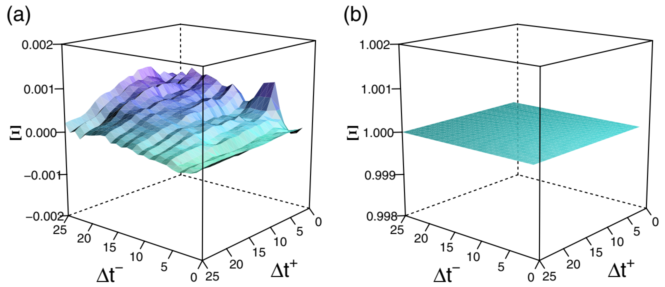


Fig. 5: (Color online) (a) Pattern conformity $\Xi_{\chi=10}(\Delta t^-, \Delta t^+)$ for a random walk time series with $3\times10^6$ time steps and $\hat{\tau}=10^4$. It is close to 0 for all combinations of $\Delta t^+$ and $\Delta t^-$. (b) Pattern conformity with the same parameter settings for a time series of a straight line, which is exactly 1 for all parameter combinations. This reflects the perfect correlation of the underlying process.

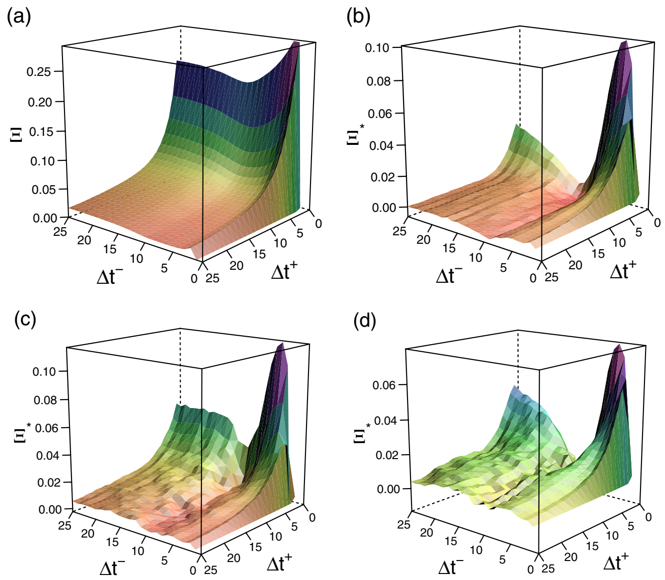


Fig. 6: (Color online) (a) Pattern conformity $\Xi^{\mathrm{FDAX}}_{\chi=100}(\Delta t^-, \Delta t^+)$ of FDAX time series with $\hat{\tau}=10^4$. (b) FDAX pattern conformity corrected by the ACRW with $\gamma=0.16$ and with $3\times10^6$ time steps. Thus, $\Xi^*=\Xi^{\mathrm{FDAX}}_{\chi=100}-\Xi^{\mathrm{ACRW}}_{\chi=100}$ is shown. (c) Identical to (b), but the fit quality of a pattern is not only calculated by the prices. Appropriate transaction volumes are incorporated, too (see text). (d) Same as (b), but inter-trade waiting times are used in combination with prices to calculate the fit quality.

contrast to cases in which one of the reference pattern and comparison pattern sequences is above and the other one below the reference point.

With this element, one can consequently define an observable for the pattern conformity, which is normalized, by

$$\Xi_\chi(\Delta t^+, \Delta t^-) = \frac{\sum_{\hat{t}=\Delta t^-}^{T-\Delta t^+}\sum_{\tau=\tau^*}^{\hat{t}} \frac{\mathrm{sgn}\left(\omega_{\hat{t}}^{\Delta t^-}(\tau,\Delta t^+)\right)}{\exp\left(\chi Q_{\hat{t}}^{\Delta t^-}(\tau)\right)}}{\sum_{\hat{t}=\Delta t^-}^{T-\Delta t^+}\sum_{\tau=\tau^*}^{\hat{t}} \frac{|\mathrm{sgn}\left(\omega_{\hat{t}}^{\Delta t^-}(\tau,\Delta t^+)\right)|}{\exp\left(\chi Q_{\hat{t}}^{\Delta t^-}(\tau)\right)}}, \quad (4)$$

where $\mathrm{sgn}(x)$ is the sign function with $\mathrm{sgn}(0)=0$. We limit the evaluation for each pattern to a maximum of $\hat{\tau}$ historical patterns in order to save computing time. Thus, the definition

$$\tau^* = \begin{cases} \hat{t}-\hat{\tau}, & \text{if } \hat{t}-\hat{\tau}-\Delta t^- \geqslant 0, \\ \Delta t^-, & \text{else,} \end{cases} \quad (5)$$

is applied. The exponential function with parameter $\chi$ weighs terms according to their degree of conformity with the reference pattern given through the fit quality $Q_{\hat{t}}^{\Delta t^-}(\tau)$.

In fig. 5, the pattern conformity for a standard random walk time series is shown, which exhibits no correlations by construction. The pattern conformity for a perfectly correlated time series —a straight line— is shown, too. With this method, it is possible to search for complex correlations in financial market data quantified through pattern conformity. In fig. 6a, $\Xi_\chi(\Delta t^-, \Delta t^+)$ is shown for the FDAX time series, in which a significant pattern conformity can be detected. Parts of the correlations stem from the trivial negative autocorrelation for $\Delta t=1$ caused by the jumps around the spread. In order to try to correct for this, in fig. 6b, the pattern conformity of the ACRW with $\gamma=0.16$ is substracted from the data of fig. 6a. Obviously, the autocorrelation for $\Delta t=1$ which is understood from the order book structure is not the sole reason for the pattern conformity shown in fig. 6a. Thus, clear evidence is obtained that financial market data show pattern correlation on very short time scales beyond the simple anti-persistence due to the gap between bid and ask prices.

So far, the comparison between reference and past patterns was based on the price time series, $Q_{\hat{t}}^{\Delta t^-}(\tau)=Q_{\hat{t}}^{p,\Delta t^-}(\tau)$, alone. Now, we also incorporate the time series of transaction volumes $v(t)$, *i.e.*, $Q_{\hat{t}}^{\Delta t^-}(\tau)=Q_{\hat{t}}^{p,\Delta t^-}(\tau)+Q_{\hat{t}}^{v,\Delta t^-}(\tau)$, to improve the pattern selection. Then the pattern conformity is increased as shown in fig. 6c. In contrast, using the inter-trade waiting time $\iota(t)$ ($Q_{\hat{t}}^{\Delta t^-}(\tau)=Q_{\hat{t}}^{p,\Delta t^-}(\tau)+Q_{\hat{t}}^{\iota,\Delta t^-}(\tau)$) decreases the pattern conformity for small values of $\Delta t^-$ as one can see in fig. 6d. These results are qualitatively independent of the applied weighting method. If the exponential weighing of terms in eq. (4) is replaced, *e.g.*, through a cutoff rule for choosing terms, comparable results are achieved. It is only important that patterns with better agreement to historical patterns have also a higher weight.

However, also an alternative concept can be used in order to reduce the trivial autocorrelation parts from the pattern conformity observable which are caused by the jumps around the bid-ask spread. But for this approach, another kind of data set is required. So far, a historical FDAX data set with transaction price, transaction time stamp and transaction volume was used. Now, we apply so-called level 1 quotes, which contain not only the last traded price $p(t)$, but also information on the best

Table 1: Autocorrelation $\rho(\Delta t)$ of the various data sets for the time lags $\Delta t=1$, $\Delta t=2$, and $\Delta t=3$.

| Data set | $\Delta t=1$ | $\Delta t=2$ | $\Delta t=3$ |
|---|---|---|---|
| FDAX time series | −0.306 | −0.022 | 0.000 |
| FDAX (last price, level 1) | −0.133 | −0.017 | −0.006 |
| FDAX (midpoint, level 1) | −0.082 | 0.027 | 0.007 |

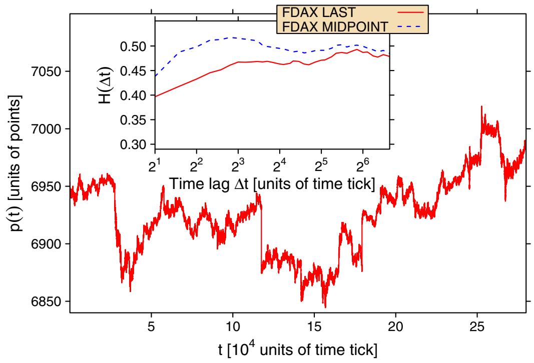


Fig. 7: (Color online) Last traded prices of FDAX level 1 data set recorded from 26 March 2007 to 30 March 2007 containing 282755 quotes. The Hurst exponent $H(\Delta t)$ in dependence of time lag $\Delta t$ calculated by the relationship $\langle|p(t+\Delta t)-p(t)|^q\rangle^{1/q}\propto\Delta t^{H_q(\Delta t)}$ with $q=2$ is shown in the inset for the time series of last traded prices and for the time series of midpoints. The midpoint time series shows a smaller anti-persistence on short time scales in conformity with a smaller absolute autocorrelation presented in table 1 at time lag $\Delta t=1$.

available bid $p_b(t)$ and the best available ask price $p_a(t)$. One has to note that time $t=1,2,\ldots,T$ is now a quote counter (in the investigations above it has to be considered as a transaction counter) with the consequence that each new best ask or best bid price causes a new quote with an old last traded price. Thus, as more bid and ask updates than updates of last price can be observed, the autocorrelation of the level 1 last price time series exhibits a smaller negative value in contrast to the original FDAX time series, as shown in table 1.

With this level 1 data set, which contains 282755 quotes, recorded from 26 March 2007 to 30 March 2007, it is possible to create a midpoint time series $p_m(t)=(p_a(t)+p_b(t))/2$, which eliminates by construction the bid-ask spread jumps. A smaller negative anticorrelation at time lag $\Delta t=1$ and positive autocorrelation at time lag $\Delta t=2$ can be observed as shown in table 1. Additionally, the scaling behavior of the time series based on last prices and the time series based on midpoints can be seen in fig. 7.

Consequently, the midpoint time series exhibits a smaller antipersistence in comparison with the last price time series. The qualitative difference between the time-dependent Hurst exponent of the last price time series and our original price time series can be explained

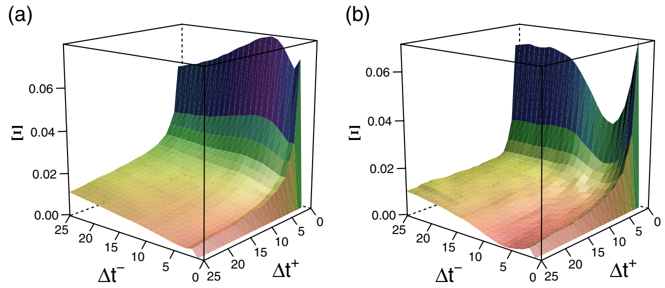


Fig. 8: (Color online) (a) Pattern conformity $\Xi^{\mathrm{FDAX}}_{\chi=10}(\Delta t^-,\Delta t^+)$ of FDAX level 1 time series for midpoint quotes. (b) Pattern conformity $\Xi^{\mathrm{FDAX}}_{\chi=100}(\Delta t^-,\Delta t^+)$ of FDAX level 1 time series for midpoint quotes. In all cases $\hat{\tau}=10^4$ is applied.

by the fact, that not necessarily each new quote contains a new last traded price. The fluctuations on medium and long time scales are due to the limited data set.

Now, the pattern conformity observable of the midpoint time series is determined for $\chi=10$ and $\chi=100$. The results are shown in fig. 8. First of all, note that an increase of $\chi$, which reflects a stricter pattern selection, leads to a broader and more distinct minimum for medium reference pattern interval length with about $\Delta t^-=10$. However, the large pattern conformity value for larger values of $\Delta t^-$ remains unaltered, if comparing fig. 8a with fig. 8b. But more interesting is the comparison of the results of fig. 8b with fig. 6b. The change from last prices to midpoint prices removes also trivial autocorrelation parts in the pattern conformity observable, and thus both data sets are in rather good agreement.

**Conclusion and outlook.** – Summarizing, we have analyzed high-frequency asset returns in detail. First of all, market impacts were investigated systematically by using conditional probability distribution functions (CPDF). The negative autocorrelation of the return time series for consecutive ticks results in a reversion tendency after a price shift on short time scales. Thus, the CPDF behavior can be reproduced qualitatively by a synthetically anti-correlated random walk, which reflects the short-time antipersistence of the Hurst exponent. Furthermore, we have introduced a method to measure complex correlations within a time series by pattern conformity. This pattern conformity observable is 0 for a random walk and 1 for a perfectly correlated time series. Checking the pattern conformity of financial data sets, we find that there is a small tendency to follow historic patterns on very short time scales. An increase of observed correlations occurs if the trading volume is included in the measure of agreement between actual and comparison time series. An additional concept for removing trivial autocorrelation parts from the pattern conformity observable was applied based on level 1 quote data sets. A further analysis in order to find complex correlations in financial markets will be undertaken in the context of prevalent market models as [18,19], because the pattern conformity can also be a foundation for the improvement of such models. Furthermore, also other noisy data sets like climate change data sets or geographical landscape data sets can be investigated.

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