ECONOPHYSICS IN A NUTSHELL

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Econophysics or "physics of financial markets" forms the interdisciplinary interface between the two disciplines economics[a] and physics[b] both terms stem from Ancient Greek. The term *econophysics* was coined by H. E. Stanley[c] in the 1990s.

The "experimental basis" of this interdisciplinary science *econophysics* is given by time series which can be used in their raw form or from which one can derive observables. Time series can be considered to be a link between economic and financial processes on the one hand and empirical analysis and physics based modeling on the other hand. Such historical price curves can be understood as a macroscopic variable for underlying microscopic processes. The price fluctuations are produced by the superposition of individual actions of market participants, thereby generating cumulative supply and demand for a traded asset—e.g. a stock. The analog in statistical physics is the emergence of macroscopic properties, caused by microscopic interactions among involved subunits.

A few decades ago, M. F. M. Osborne[d] [4] and B. B. Mandelbrot[e] [5] analyzed time series related to financial markets realizing the non-Gaussian shape of the price change distributions[6]—a Gaussian distribution is still an often used assumption for economic models. With these pioneering findings, they became early fathers of *econophysics*. Mandelbrot used for his analysis historical cotton times and sales records dating back to the beginning of the 20th century. His findings indicated that financial market time series obey a complex statistical behavior which has similarities with the non-Gaussian properties of critical fluctuations in physics[7–9]. The studies of Mandelbrot were based on data sets of very limited length. Alongside technological progress in information technology, trading processes in international financial markets were adapted and fully electronic trading platforms have been established. Thus, a gargantuan amount of historical financial time series are available for researchers and practitioners with an extremely high time resolution[f]. A century ago, only daily data were accessible, whereby the situation today is such that transaction records are available on time scales of milliseconds. The continuously increasing number of transaction records confirm the results obtained by Mandelbrot. Furthermore, the use of Gaussian approximations in finance has become more and more questionable.

However, the first work in this area which can be related to current *econophysics* was performed much earlier as reported for example in[9]. In 1900, a young PhD student of H. Poincaré[g] finished his thesis. His name was L. Bachelier[a]. On the one hand, Poincaré lauded the brilliant

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[a] *Ancient Greek*: οἰκονομία — housekeeping, management.

[b] *Ancient Greek*: φυσική τέχνη — art of handling the nature.

[c] **Harry Eugene Stanley** (*28 March 1941 in Oklahoma City, USA): an American physicist and University Professor at Boston University. He has made seminal contributions to statistical physics and was one of the pioneers of interdisciplinary science[1].

[d] **Maury F. M. Osborne**: Physicist. His publication "Brownian Motion in the Stock Market"[2] marks the starting point in literature about the geometric Brownian motion in a financial mathematics context.

[e] **Benòît B. Mandelbrot** (*20 November 1924 in Warsaw, Poland): French and American mathematician who is known as the father of fractal geometry. He applies fractal geometry to various scientific fields[1].

[f] In August 2009, the Deutsche Börse Group reduced the network latency between Frankfurt and London to below 5 milliseconds, setting new standards in the industry[3].

[g] **Jules Henri Poincaré** (*29 April 1854 in Nancy, France; †17 July 1912 in Paris, France): French mathematician, theoretical physicist, and a philosopher of science[1].

way in which the Gaussian distribution was derived. On the other hand, he pointed out that the subject of his thesis entitled "Théorie de la spéculation"[10] was completely different from subjects of other theses. Bachelier proposed the *random walk* as model for price fluctuations in his PhD thesis. Thus, he developed the mathematics of Brownian motion with his description of financial market processes—five years before the famous publication of A. Einstein[b] on Brownian motion appeared in 1905[11]. Until the early 1940s Bachelier's work tended to be sidelined. But living in the shadows is not exceptional for revolutionary scientific contributions as exemplified by the famous publication by E. Ising[c 12] about a model which has become a standard model in statistical physics today—the Ising model[d]. W. Heisenberg[e] cited this work in 1928. The breakthrough of the Ising model coincided with the exact solution for the Ising model on a two-dimensional square lattice achieved by L. Onsager[f] in 1944[13]. The Ising model can not only be used to describe ferromagnetism as originally intended; it can also be extended to describe more complex magnetic systems, such as spin glasses, in which competing interactions between various spins lead to frustration effects and slow ("glassy") dynamics. The energy landscapes of such systems exhibit a vast amount of local minima. The task is to find those spin configurations for which the energy is minimal. The inverse problem, namely the calculation of interactions for the proposed spin configurations which should form local minima in the energy landscape, is known as the Hopfield model for neural networks[g]. Today, the Ising model is used for various—ordered and disordered—complex systems in many areas of science. The various applications also include the interdisciplinary field of sociophysics[17] in which social interactions are treated by a modification of the Ising model—the Sznajd model[18]. In this way, it is possible to investigate, for example, the stability of democracy in the German Federal State of Bavaria[19] based on membership levels of the dominating political parties. Many further interdisciplinary streams of research in which physics' concepts and methods are used have appeared. One example is the physics of transport and pedestrians. An often asked question is how spontaneous traffic jams occur[20,21]. Beside D. Helbing[h], who is one of the leading researchers in this field, one prominent scientist in this field of research is D. Brockmann[i]. His research on complex systems spans from transportation networks to infectious diseases. He has developed computational models, new analytic and numerical techniques, and large-scale quantitative and predictive computer simulations for studying various aspects of the dynamics of epidemics. For example, he has used data from www.wheresgeorge.com—a website where users enter the serial numbers of their dollar bills—in order to track their travels[22]. In this way, patterns and laws of human mobility could be identified. From that information, Brockmann was able to reconstruct a multi-scale human mobility network of the U.S. including small scale daily commuter traffic, intermediate traffic, and long distance air travel. Based on this mobility network, Brockmann has modeled how diseases spread throughout the country. He and his research group have also created a map of large scale community boundaries in the United States, which sometimes differs from administrative boundaries. These effective maps show that some states, like Missouri or Pennsylvania, are essentially cut into halves. Other boundaries coincide with geographic features, such as the Appalachian Mountains. Brockmann also develops models for disease spreading via transportation networks, in order to quantify the susceptibility of various regions to epidemic threats and to develop more efficient containment strategies.

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[a]**Louis Jean-Baptiste Alphonse Bachelier** (*11 March 1870 in Le Havre, France; †26 April 1946 in St-Servan-sur-Mer, France): French mathematician who is credited with being the first person to model the stochastic process which is now called Brownian motion[1].

[b]**Albert Einstein** (*14 March 1879 in Ulm, Germany; †18 April 1955 in Princeton, USA): German-born Swiss-American theoretical physicist, philosopher and author who is widely regarded as one of the most influential and best known scientists and intellectuals of all time. He is often regarded as the father of modern physics[1].

[c]**Ernst Ising** (*10 May 1900 in Cologne, Germany; †11 May 1998 in Peoria, USA): German physicist and mathematician who is best remembered for the development of the Ising model[1]. The Ising model describes the transition of an unordered paramagnetic material at high temperatures to a ferromagnetic material below a critical temperature—the Curie temperature T_C.

[d]Recently, we demonstrated how to accelerate Monte Carlo simulations of the 2D and 3D Ising model on graphics card architectures[14,15].

[e]**Werner Heisenberg** (*5 December 1901 in Würzburg, Germany; †1 February 1976 in Munich, Germany): Norwegian-born American physical chemist and theoretical physicist. He obtained the Nobel Prize in Physics in 1932[1].

[f]**Lars Onsager** (*27 November 1903 in Kristiania, Norway; †5 October 1976 in Coral Gables, USA): Norwegian-born American physical chemist and theoretical physicist. He obtained the Nobel Prize in Chemistry in 1968[1].

[g]In a spin glass, the interactions between individual spins are fixed. It is the aim—and sometimes a challenge—to find the ground state in the energy landscape. In a Hopfield network the configurations, which correspond to the ground states, are given and one has to find the interactions[16].

[h]**Dirk Helbing**: professor of sociology, in particular of modeling and simulation, at ETH Zurich since 2007 and external professor of the Santa Fe Institute since 2009.

[i]**Dirk Brockmann**: associate professor of engineering sciences and applied mathematics at the McCormick school of engineering and applied science at Northwestern University. He received the Young Scientist Award 2010 for socio- and econophysics from the physics of socio-economic systems division of the German Physical Society.

Bachelier did not gain such publicity in the first years after his work was published. Applying his ideas to describe French bonds took significant time. At that time, his interdisciplinary work was not popular in the scientific community at all—today, there actually exists a "Bachelier Finance Society"[a]. However, in the meantime it turned out that the work of A. Kolmogorov[b] was inspired by Bachelier's results as can be retraced in[23]. In 1944—almost half a century later—Itô[c] used Bachelier's PhD thesis as motivation for his famous calculus, now called Itô calculus. Later, Osborne introduced geometric Brownian motion which is an extension of Brownian motion. It is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion. This is advantageous in financial market modeling since the logarithm can only take positive values—a meaningful assumption for the prices of stocks. This compatibility with the axioms of economics was shown by P. A. Samuelson[d] for which he obtained the Nobel Prize in Economics in 1970. Asset prices could be assumed to be log-normally distributed. This implied normally distributed returns.

Based on that, F. Black[e] and M. Scholes[28]—as well as R. Merton[f] [29] independently of both of them—developed a theory for evaluating stock options using geometric Brownian motion. This theory had a huge impact on financial markets. Up to that time, it had not been possible to obtain an "objective" valuation of options. Thus, the transaction numbers and transaction volumes of option contracts increased rapidly after the introduction of this theory. Today, modern *risk management* is inconceivable without options. In 1997, the Nobel Prize in Economics was awarded to Scholes and Merton for their work. However, it must also be mentioned that this theory, now known as the Black Scholes model, is accompanied by problematic aspects. Continuous time trading is a first assumption which is matched most closely by foreign exchange markets[g] but which is not fulfilled by other exchange platforms. The second requirement is a continuous price path which is unrealistic—this is already contradicted by the existence of opening gaps. An opening gap is a direct consequence of non-continuous trading. At the beginning of an exchange trading day a stock does not necessarily start being traded at the same price level as the day before. The difference between opening price and closing price one day before is known as the opening gap. If there is for example an important news release in the period of time without trading, larger opening gaps occur. In addition to these two assumptions, it is critical to note that the important input variable, the volatility (annualized standard deviation) of future stock returns, is not known in advance. Furthermore, the Black Scholes model is based on normally distributed asset returns. This aspect contradicts earlier findings mentioned in this section: thus, economics uses simplifying assumptions which were already falsified by Mandelbrot in 1963. Several papers confirm his findings and, most importantly, H. E. Stanley et al. were able to quantify the tails of the return distributions. They found that the tails are consistent with cubic power laws[8,30,31].

In this context, physicists used a wide range of methods in trying to examine the discrepancy between theory and the reality of economic and financial markets. They applied physical methods to the "many particle system" made up by financial markets. Non-trivial features of financial market time series are called *empirical stylized facts*.

It is on the basis of commonly accepted facts that physicists started to model financial markets. Simple models were proposed as a result, for example at the end of

[a]More information can be found on www.bachelierfinance.org.

[b]**Andrey Nikolaevich Kolmogorov** (*25 April 1903 in Tambow, Russia; †20 October 1987 in Moscow, Russia): one of the most famous mathematicians of the 20th century[1].

[c]**Itô Kiyoshi** (*7 September 1915 in Hokusei-chô, Japan; †10 November 2008 in Kyôto, Japan): a Japanese mathematician whose work is now called Itô calculus. It facilitates mathematical understanding of random events. His theory is widely applied, for instance in financial mathematics[1].

[d]**Paul Anthony Samuelson** (*15 May 1915 in Gary, USA; †13 December 2009 in Belmont, USA): an American economist, and the first American to win the Nobel Prize in Economics. The Swedish Royal Academies stated, when awarding the prize, that he "has done more than any other contemporary economist to raise the level of scientific analysis in economic theory"[1].

[e]**Fischer Sheffey Black** (*11 January 1938 in Washington, USA; †30 August 1995 in New York, USA): an American economist, best known as one of the authors of the famous *Black-Scholes equation*[1].

[f]**Myron Samuel Scholes** (*1 July 1941 in Timmins, Canada) and **Robert Cox Merton** (*31 July 1944 in New York, USA) were among the board of directors of Long-Term Capital Management (LTCM), a hedge fund that failed spectacularly in 1998 after losing USD 4.6 billion in less than four months. The Federal Reserve was so concerned about the potential impact of LTCM's failure on the financial system that Alan Greenspan arranged for a group of 19 banks and other firms to provide sufficient liquidity for the banking system to survive. Although these investors were eventually paid off, the reputations of Merton and Scholes were tarnished. LTCM used trading strategies such as fixed income arbitrage, *statistical arbitrage*, and *pairs trading*, combined with high leverage. *Arbitrage* is the practice of taking advantage of a price difference between two or more markets[10].

[g]The foreign exchange market (*FOREX*) is a worldwide decentralized over-the-counter financial market for the trading of currency pairs. There is a continuous trading established—an exception marks the weekend. The average daily trading volume of this non-regulated, global market was, e.g., roughly USD 1.9 trillion in April 2004[24].

[h]This early contribution of physicists of modeling financial markets using agent based models is based on a reaction diffusion process[25]. It is able to reproduce some *empirical stylized facts*—e.g., non-trivial Hurst exponents and non-Gaussian price change distributions. Large price changes occur more frequently than predicted by the Gaussian distribution. A few years later, a model was published[26,27] incorporating two different order types—limit orders and market orders. A model with an additional

the last century[25,8]. A simple agent-based model[h] [25] uses imitation and feedback. It is able to reproduce simple examples of the group of *empirical stylized facts*. In recent years, physicists started to investigate and understand the price formation process in detail on a microscopic level as simple models were not able to completely reproduce the behavior of financial markets. In this context, the statistical model of the continuous double auction[32,33] stand out. The continuous double auction is used for organizing trading processes at electronic financial exchanges. An overview of agent-based models and empirical stylized facts can be found in[34].

The study of financial markets and complex systems in general will be continued. Recently, we characterized trend switching processes in financial markets[35–37] and analyzed fluctuation patterns in financial time series[37,38]. ❐

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rule set was developed in 2003[39]. There, a Poisson process is applied for the cancellation of submitted limit orders. In recent years, further agent based market models[39–43] have been proposed in order to reproduce further *empirical stylized facts*.

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